FUNDAMENTOS DE TRIGONOMETRIA PLANA

Triangulo: poligono de tres lados, y por tanto tres angulos.
TIPOS :
Tres lados iguales, triangulo equilatero
Dos lados iguales, triangulo isosceles
Ningun lado igual triangulo escaleno

LOS TRES ANGULOS DE UN TRIANGULO SIEMPRE SUMAN 180º

Triangulo rectangulo: un angulo mide 90º   Por lo tanto la suma de los otro dos tambien son 90º

NOMBRES:
Lado OPUESTO al angulo de 90º   HIPOTENUSA
Los otros dos lados, CATETOS

Longitud de los lados

TEOREMA DE PITAGORAS

$h^2 = a^2 + b^2$ siendo h la hipotenusa y a y b los catetos. De esta formula resulta

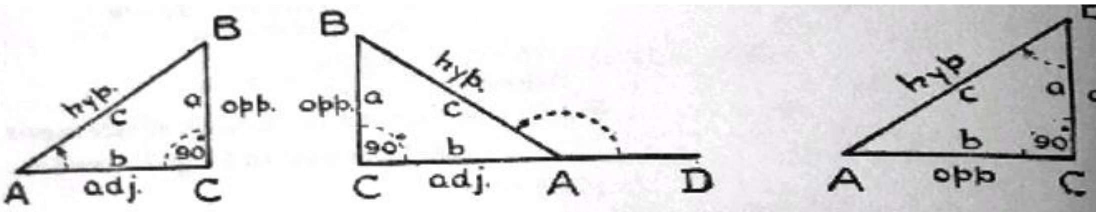
$$h = \sqrt{a^2 + b^2}$$

La trigonometria estudia la relacion entre angulos y lados.

Figura:
Triangulo ABC referido al angolo A
lado BC opuesto angulo A
AC contiguo A
AB hipotenusa

Referido al angulo B
AC opuesto a B
BC contiguo a B
AB hipotenusa

Seno Fig 1



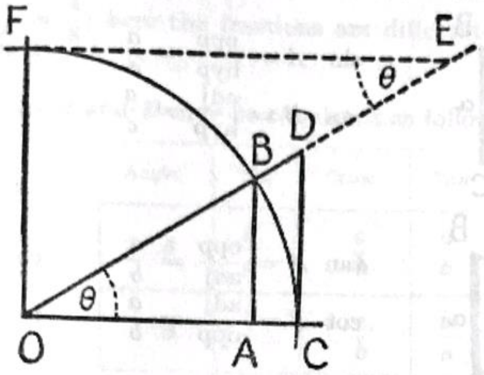
In triangle ABC, where angle A is referred to, either internal or external, then

Side BC is **opposite to the angle** (opp.)
Side AC is **adjacent to the angle** (adj.)
Side AB is the **hypotenuse** (hyp.)

But if angle B is referred to then

Side AC is **opposite to the angle** (opp.)
Side BC is **adjacent to the angle** (adj.)
Side AB is the **hypotenuse** (hyp.)

Funciones trigonometricas
Fig 2



$$\sin\theta = AB = \frac{AB}{\text{a radius}} = \frac{AB}{OB} = \frac{\text{opp}}{\text{hyp}}$$

$$\tan\theta = CD = \frac{CD}{\text{a radius}} = \frac{CD}{OC} = \frac{\text{opp}}{\text{adj}}$$

$$\sec\theta = OD = \frac{OD}{\text{a radius}} = \frac{OD}{OC} = \frac{\text{hyp}}{\text{adj}}$$

$$\cos\theta = OA = \frac{OA}{\text{a radius}} = \frac{OA}{OB} = \frac{\text{adj}}{\text{hyp}}$$

$$\cot\theta = FE = \frac{FE}{\text{a radius}} = \frac{FE}{OF} = \frac{\text{adj}}{\text{opp}}$$

$$\text{cosec}\,\theta = OE = \frac{OE}{\text{a radius}} = \frac{OE}{OF} = \frac{\text{hyp}}{\text{opp}}$$

Sen $\theta = AB$ cos $\theta = OA$
Tan $\theta = CD$ sec $\theta = OD$

Cotg $\theta = FE$ cosec $\theta = OE$

La unidad de medida es el radio del circulo, que se considera 1
Se establecen las siguientes relaciones fundamentales:

$$sen\ \theta = AB = \frac{AB}{a\ radio} = \frac{AB}{OB} = \frac{opuesto}{hipot}$$

$$\tan\ \theta = CD = \frac{CD}{a\ radio} = \frac{CD}{OC} = \frac{opuesto}{contiguo}$$

$$\sec\ \theta = OD = \frac{OD}{a\ radio} = \frac{OD}{OC} = \frac{hipot}{contiguo}$$

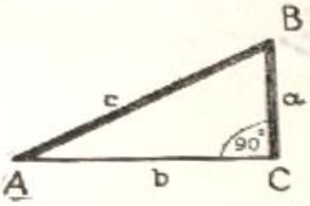
$$\cos\ \theta = OA = \frac{OA}{a\ radio} = \frac{OA}{OB} = \frac{contiguo}{hipot}$$

$$\cot g\ \theta = FE = \frac{FE}{a\ radio} = \frac{FE}{OF} = \frac{contiguo}{opues}$$

$$\text{cosec}\ \theta = OE = \frac{OE}{a\ radio} = \frac{OE}{OF} = \frac{hipot}{opues}$$

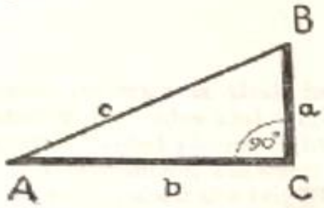
EJERCICIOS

In naming the sides of a triangle it is often convenient to name the side opposite $\angle A$ as "*a*," the side opposite $\angle B$ as "*b*" and the side opposite $\angle C$ as "*c*."



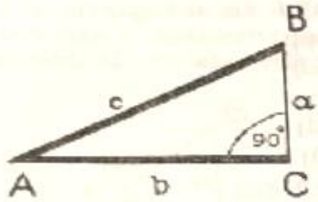
$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} \qquad \text{cosec } A = \frac{\text{hyp}}{\text{opp}} = \frac{c}{a}$$

$$\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{a}{c} \qquad \sec B = \frac{\text{hyp}}{\text{adj}} = \frac{c}{a}$$



$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{a}{b} \qquad \cot A = \frac{\text{adj}}{\text{opp}} = \frac{b}{a}$$

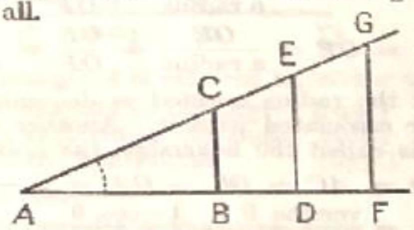
$$\cot B = \frac{\text{adj}}{\text{opp}} = \frac{a}{b} \qquad \tan B = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}$$



$$\sec A = \frac{\text{hyp}}{\text{adj}} = \frac{c}{b} \qquad \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c}$$

$$\text{cosec } B = \frac{\text{hyp}}{\text{opp}} = \frac{c}{b} \qquad \sin B = \frac{\text{opp}}{\text{hyp}} = \frac{b}{c}$$

In the following figure there are three similar triangles, ABC, ADE and AFG, angle A being common to all.



$$\sin A = \frac{BC}{CA} = \frac{DE}{EA} = \frac{FG}{GA} = \frac{\text{opp}}{\text{hyp}} \text{ in every case}$$

$$\text{cosec } A = \frac{AC}{CB} = \frac{AE}{ED} = \frac{AG}{GF} = \frac{\text{hyp}}{\text{opp}} \quad "$$

$$\tan A = \frac{CB}{AB} = \frac{DE}{DA} = \frac{FG}{FA} = \frac{\text{opp}}{\text{adj}} \quad "$$

$$\cot A = \frac{AB}{BC} = \frac{AD}{DE} = \frac{AF}{FG} = \frac{\text{adj}}{\text{opp}} \quad "$$

$$\sec A = \frac{AC}{AB} = \frac{AE}{AD} = \frac{AG}{AF} = \frac{\text{hyp}}{\text{adj}} \quad "$$

$$\cos A = \frac{AB}{AC} = \frac{AD}{AE} = \frac{AF}{AG} = \frac{\text{adj}}{\text{hyp}} \quad "$$

Relaciones reciprocas

El reciproco de ⅖ es 5/2
El reciproco de x/y es y/x
Una cantidad multiplicada por su reciproca, es 1

Por ej ⅖ x 5/2 = 1

$\frac{x}{y}$x $\frac{y}{x}$=1

Refiriendonos al triangulo ABC tenemos que ang C=90º

$$sen\ A = \frac{a}{c}\ y\ \text{cosec}\ A = \frac{c}{a}\ pero\ \frac{a}{c}x\ \frac{c}{a} = 1$$

$$por\ \tan to\ sen\ A\ x\ \text{cosec}\ A = 1$$

$$\tan A = \frac{a}{b} \quad , \quad \cot A = \frac{b}{a} \quad por\ lo\ que \quad \tan\ y\ \cot\ son\ reciproc*$$

$$\sec A = \frac{c}{b} \quad , \quad \cos A = \frac{b}{c} \quad \sec\ y\ \text{cosec}\ son\ reciprocas$$

$$ejemplo$$

$$\frac{x}{senA} = x\ .\ \text{cosec}\ A \quad ; \quad \frac{x}{\cot A} = x\ \tan A \quad : \quad \frac{x}{\cos A} = x\ \sec A$$

$$no\ hay\ que\ confundir\ RECIPROCO\ con\ COMPLEMENTARIO$$

$$reciproco \qquad\qquad\qquad complemento$$

$$senA = \frac{1}{\text{cosec}\ A} = \cos\ (90º - A)$$

$$\tan\ A = \frac{1}{\cot A} = \cot(90º - A\ )$$

$$\sec A = \frac{1}{\cos A} = \text{cosec}(90º - A\ )$$

RELACIONES DE COMPLEMENTARIOS:

Ejemplos:

$$probar\ que$$

$$\cos(90º - A) = senA$$

$$A + B + C = 180º$$

$$sustraer\ C = 90º$$

$$A + B = \qquad 180º$$

$B = (90^o - A)$

$Si\ \ (90^o - A) = B$

$\cos(90 - A) = \cos B = \frac{a}{c} = senA$

$Ejemplo\ 2$

$\Pr obar\ que$

$\tan(90 - A) = \cot A$

$(90 - A) = B\ \ por\ lo\ que \quad \tan(90 - A) = \tan B = \frac{b}{a} = \cot\ \ A$

RECIPROCOS:

$\Pr obar\ que$

$senA \operatorname{cosec} A = 1$

$refiriendonos\ al\ triangulo\ ABC\ \ C = 90^o$

$senA \operatorname{cosec} A = \frac{a}{c}\ \frac{c}{a} = 1$

$(ii)\ \ probar\ que\ \ \sec A \cos A = 1$

$\sec A \cos A = \frac{c}{b}\ \frac{b}{c} = 1$

Relaciones basicas

triangulo ABC en el que $C = 90º$

$\tan A = \frac{a}{b}$ *divi* dim *os numerador y deno* min *ador por los res* tan *tes lados del triangulo*;

$$(i)\ \tan A = \frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{senA}{\cos A}$$

$$(ii) \cot A = \frac{b}{a} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{\cos A}{senA}$$

$$(iii)\ senA = \frac{a}{c} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{\tan A}{\sec A}$$

$$(iv)\ \sec A = \frac{c}{b} = \frac{\frac{c}{a}}{\frac{b}{a}} = \frac{\text{cosec } A}{\cot A}$$

Igualmen

cosA= cotA/cosecA

cosecA=secA/tanA

IMPORTANTE MEMORIZAR:

$$\tan A = \frac{senA}{\cos A}$$

$$\cot A = \frac{\cos A}{senA}$$

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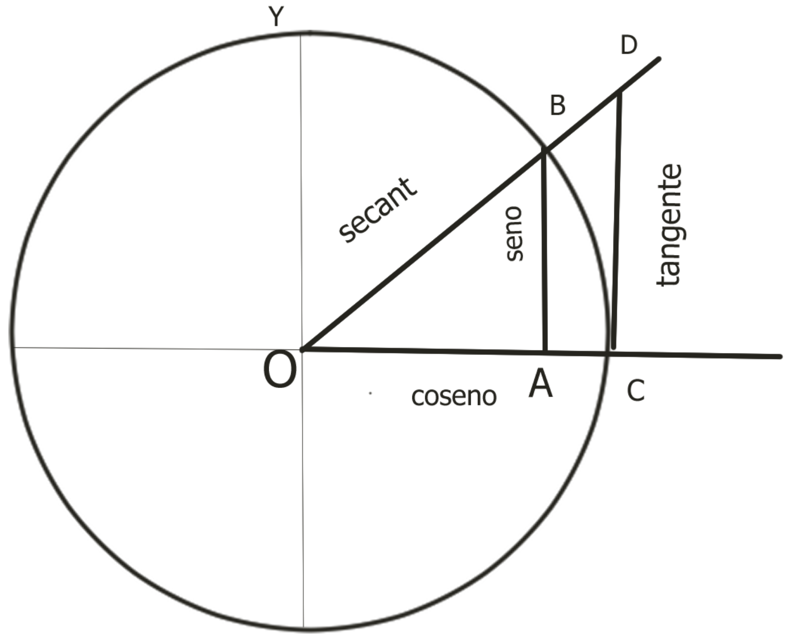
FUNCIONES TRIGINIMETRICAS DEL ANGULO 0º


Y
D
B
secant
seno
tangente
O
A
C
coseno